

Rubi 4.16.1.4 Integration Test Results

on the problems in the test-suite directory "4 Trig functions"

Test results for the 538 problems in "4.1.0 (a sin)^m (b trg)^{n.m}"

Test results for the 348 problems in "4.1.10 (c+d x)^m (a+b sin)^{n.m}"

Test results for the 72 problems in "4.1.1.1 (a+b sin)^{n.m}"

Test results for the 113 problems in "4.1.11 (e x)^m (a+b xⁿ)^p sin.m"

Test results for the 357 problems in "4.1.12 (e x)^m (a+b sin(c+d xⁿ))^{p.m}"

Test results for the 653 problems in "4.1.1.2 (g cos)^p (a+b sin)^{m.m}"

Problem 648: Result valid but suboptimal antiderivative.

$$\int (e \cos[c + d x])^{-3-m} (a + b \sin[c + d x])^m dx$$

Optimal (type 5, 311 leaves, ? steps):

$$\begin{aligned} & \frac{(e \cos[c + d x])^{-m} \sec[c + d x]^4 (-1 + \sin[c + d x]) (1 + \sin[c + d x]) (a + b \sin[c + d x])^{1+m}}{(a - b) d e^3 (2 + m)} + \frac{1}{(a - b)^2 d e^3 m (2 + m)} \\ & \frac{(-2 b + a (2 + m)) (e \cos[c + d x])^{-m} \sec[c + d x]^4 (-1 + \sin[c + d x]) (1 + \sin[c + d x])^2 (a + b \sin[c + d x])^{1+m} -}{(a - b)^3 d e^3 m (1 + m)} \\ & \frac{1}{(-b^2 + a^2 (1 + m))} (e \cos[c + d x])^{-m} \text{Hypergeometric2F1}\left[\frac{m}{2}, 1 + m, 2 + m, -\frac{2 (a + b \sin[c + d x])}{(a - b) (-1 + \sin[c + d x])}\right] \\ & \sec[c + d x]^4 (1 + \sin[c + d x])^3 \left(\frac{(a + b) (1 + \sin[c + d x])}{(a - b) (-1 + \sin[c + d x])}\right)^{\frac{1}{2} (-2 + m)} (a + b \sin[c + d x])^{1+m} \end{aligned}$$

Result (type 5, 420 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{(e \cos[c + d x])^{-2-m} (a + b \sin[c + d x])^{1+m}}{(a - b) d e (2 + m)} - \\
 & \left(b (e \cos[c + d x])^{-2-m} \text{Hypergeometric2F1}\left[1 + m, \frac{2 + m}{2}, 2 + m, \frac{2 (a + b \sin[c + d x])}{(a + b) (1 + \sin[c + d x])}\right] (1 - \sin[c + d x]) \left(-\frac{(a - b) (1 - \sin[c + d x])}{(a + b) (1 + \sin[c + d x])}\right)^{m/2} \right. \\
 & \left. (a + b \sin[c + d x])^{1+m}\right) / ((a^2 - b^2) d e (1 + m) (2 + m)) + \frac{a (e \cos[c + d x])^{-2-m} (1 + \sin[c + d x]) (a + b \sin[c + d x])^{1+m}}{(a^2 - b^2) d e (2 + m)} + \\
 & \left(2^{-m/2} a (a + b + a m) (e \cos[c + d x])^{-2-m} \text{Hypergeometric2F1}\left[-\frac{m}{2}, \frac{2 + m}{2}, \frac{2 - m}{2}, \frac{(a - b) (1 - \sin[c + d x])}{2 (a + b \sin[c + d x])}\right] \right. \\
 & \left. (1 - \sin[c + d x]) \left(\frac{(a + b) (1 + \sin[c + d x])}{a + b \sin[c + d x]}\right)^{\frac{2 + m}{2}} (a + b \sin[c + d x])^{1+m}\right) / ((a - b) (a + b)^2 d e m (2 + m))
 \end{aligned}$$

Test results for the 36 problems in "4.1.13 (d+e x)^m sin(a+b x+c x^2)^n.m"

Test results for the 208 problems in "4.1.1.3 (g tan)^p (a+b sin)^m.m"

Test results for the 837 problems in "4.1.2.1 (a+b sin)^m (c+d sin)^n.m"

Test results for the 1563 problems in "4.1.2.2 (g cos)^p (a+b sin)^m (c+d sin)^n.m"

Problem 1479: Unable to integrate problem.

$$\int \frac{\sec[e + f x]^2 (a + b \sin[e + f x])^{3/2}}{\sqrt{d \sin[e + f x]}} dx$$

Optimal (type 4, 312 leaves, ? steps):

$$\begin{aligned}
& \frac{\sec(e+fx) (b + a \sin(e+fx)) \sqrt{a+b \sin(e+fx)}}{f \sqrt{d \sin(e+fx)}} - \\
& \frac{(a+b)^{3/2} \sqrt{-\frac{a(-1+\csc(e+fx))}{a+b}} \sqrt{\frac{a(1+\csc(e+fx))}{a-b}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b \sin(e+fx)}}{\sqrt{a+b} \sqrt{d \sin(e+fx)}}\right], -\frac{a+b}{a-b}] \tan(e+fx)}{\sqrt{d} f} - \\
& \left(b (a+b) \sqrt{-\frac{a(-1+\csc(e+fx))}{a+b}} \sqrt{\frac{b+a \csc(e+fx)}{-a+b}} \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{-\frac{b+a \csc(e+fx)}{a-b}}\right], \frac{-a+b}{a+b}] (1+\sin(e+fx)) \tan(e+fx) \right) / \\
& \left(f \sqrt{\frac{a(1+\csc(e+fx))}{a-b}} \sqrt{d \sin(e+fx)} \sqrt{a+b \sin(e+fx)} \right)
\end{aligned}$$

Result (type 8, 37 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sec(e+fx)^2 (a+b \sin(e+fx))^{3/2}}{\sqrt{d \sin(e+fx)}}, x\right]$$

Problem 1480: Unable to integrate problem.

$$\int \frac{\sec(e+fx)^4 (a+b \sin(e+fx))^{5/2}}{\sqrt{d \sin(e+fx)}} dx$$

Optimal (type 4, 366 leaves, ? steps):

$$\begin{aligned}
& \frac{5 a \sec(e+fx) (b + a \sin(e+fx)) \sqrt{a+b \sin(e+fx)}}{6 f \sqrt{d \sin(e+fx)}} + \frac{\sec(e+fx)^3 \sqrt{d \sin(e+fx)} (a+b \sin(e+fx))^{5/2}}{3 d f} - \\
& \frac{5 a (a+b)^{3/2} \sqrt{-\frac{a(-1+\csc(e+fx))}{a+b}} \sqrt{\frac{a(1+\csc(e+fx))}{a-b}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b \sin(e+fx)}}{\sqrt{a+b} \sqrt{d \sin(e+fx)}}\right], -\frac{a+b}{a-b}] \tan(e+fx)}{6 \sqrt{d} f} - \\
& \left(5 a b (a+b) \sqrt{-\frac{a(-1+\csc(e+fx))}{a+b}} \sqrt{\frac{b+a \csc(e+fx)}{-a+b}} \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{-\frac{b+a \csc(e+fx)}{a-b}}\right], \frac{-a+b}{a+b}] (1+\sin(e+fx)) \tan(e+fx) \right) / \\
& \left(6 f \sqrt{\frac{a(1+\csc(e+fx))}{a-b}} \sqrt{d \sin(e+fx)} \sqrt{a+b \sin(e+fx)} \right)
\end{aligned}$$

Result (type 8, 87 leaves, 1 step):

$$\frac{\sec[e + fx]^3 \sqrt{d \sin[e + fx]} (a + b \sin[e + fx])^{5/2}}{3 df} + \frac{5}{6} a \text{Unintegrable}\left[\frac{\sec[e + fx]^2 (a + b \sin[e + fx])^{3/2}}{\sqrt{d \sin[e + fx]}}, x\right]$$

Problem 1515: Unable to integrate problem.

$$\int \frac{\sec[e + fx]^6 (a + b \sin[e + fx])^{9/2}}{\sqrt{d \sin[e + fx]}} dx$$

Optimal (type 4, 502 leaves, ? steps):

$$\begin{aligned} & -\frac{3ab(-2a^2+b^2)\cos[e+fx]\sqrt{a+b\sin[e+fx]}}{5f\sqrt{d\sin[e+fx]}} + \\ & \frac{\sec[e+fx]^5 \sqrt{d \sin[e + fx]} (a + b \sin[e + fx])^{9/2}}{5 df} - \frac{1}{20 df} 3a \sec[e + fx]^3 \sqrt{d \sin[e + fx]} \sqrt{a + b \sin[e + fx]} \\ & (-a(7a^2+b^2) + 2b(-7a^2+b^2) \sin[e+fx] + 5a(a^2-b^2) \sin[e+fx]^2 + (8a^2b-4b^3) \sin[e+fx]^3) - \frac{1}{20 \sqrt{d} f} 3a(a+b)^{3/2}(5a^2+3ab-4b^2) \\ & \sqrt{-\frac{a(-1+\csc[e+fx])}{a+b}} \sqrt{\frac{a(1+\csc[e+fx])}{a-b}} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b} \sin[e+fx]}{\sqrt{a+b} \sqrt{d \sin[e+fx]}}\right], -\frac{a+b}{a-b}] \tan[e+fx] - \\ & \frac{1}{5 df \sqrt{a+b \sin[e+fx]}} 3b(2a^4-3a^2b^2+b^4) \sqrt{-\frac{a(-1+\csc[e+fx])}{a+b}} \text{EllipticE}[\text{ArcSin}\left[\sqrt{-\frac{b+a \csc[e+fx]}{a-b}}\right], 1-\frac{2a}{a+b}] \\ & \sqrt{d \sin[e+fx]} \sqrt{-\frac{a \csc[e+fx]^2 (1+\sin[e+fx]) (a+b \sin[e+fx])}{(a-b)^2}} \tan[e+fx] \end{aligned}$$

Result (type 8, 87 leaves, 1 step):

$$\frac{\sec[e + fx]^5 \sqrt{d \sin[e + fx]} (a + b \sin[e + fx])^{9/2}}{5 df} + \frac{9}{10} a \text{Unintegrable}\left[\frac{\sec[e + fx]^4 (a + b \sin[e + fx])^{7/2}}{\sqrt{d \sin[e + fx]}}, x\right]$$

Test results for the 51 problems in "4.1.2.3 (g sin)^p (a+b sin)^m (c+d sin)^n.m"

Test results for the 358 problems in "4.1.3.1 (a+b sin)^m (c+d sin)^n (A+B sin).m"

Test results for the 19 problems in "4.1.4.1 (a+b sin)^m (A+B sin+C sin^2).m"

Test results for the 34 problems in "4.1.4.2 (a+b sin)^m (c+d sin)^n (A+B sin+C sin^2).m"

Test results for the 594 problems in "4.1.7 (d trig)^m (a+b (c sin)^n)^p.m"

Problem 391: Unable to integrate problem.

$$\int \frac{\sec [c + d x]^2}{a + b \sin [c + d x]^3} dx$$

Optimal (type 3, 299 leaves, ? steps):

$$\begin{aligned} & \frac{2 (-1)^{2/3} b^{2/3} \operatorname{ArcTan} \left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \tan \left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{3 a^{2/3} (a^{2/3} - (-1)^{2/3} b^{2/3})^{3/2} d} - \frac{2 b^{2/3} \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 a^{2/3} (a^{2/3} - b^{2/3})^{3/2} d} + \\ & \frac{2 (-1)^{1/3} b^{2/3} \operatorname{ArcTan} \left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}} \right]}{3 a^{2/3} (a^{2/3} + (-1)^{1/3} b^{2/3})^{3/2} d} + \frac{\sec [c + d x] (b - a \sin [c + d x])}{(-a^2 + b^2) d} \end{aligned}$$

Result (type 8, 25 leaves, 0 steps):

$$\text{Unintegrable} \left[\frac{\sec [c + d x]^2}{a + b \sin [c + d x]^3}, x \right]$$

Problem 392: Unable to integrate problem.

$$\int \frac{\sec [c + d x]^4}{a + b \sin [c + d x]^3} dx$$

Optimal (type 3, 1093 leaves, ? steps):

$$\begin{aligned}
& - \frac{2 (-1)^{2/3} a^{2/3} b^{8/3} \operatorname{ArcTan} \left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} (a^2 - b^2)^2 d} - \frac{2 b^2 (2 a^2 + b^2) \operatorname{ArcTan} \left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} (a^2 - b^2)^2 d} + \frac{2 a^{2/3} b^{8/3} \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{\sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d} + \\
& \frac{2 b^2 (2 a^2 + b^2) \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d} + \frac{2 b^{4/3} (a^2 + 2 b^2) \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 \sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d} - \frac{2 (-1)^{1/3} a^{2/3} b^{8/3} \operatorname{ArcTan} \left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}} \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} (a^2 - b^2)^2 d} + \\
& \frac{2 b^2 (2 a^2 + b^2) \operatorname{ArcTan} \left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} (a^2 - b^2)^2 d} - \frac{2 b^{4/3} (a^2 + 2 b^2) \operatorname{ArcTanh} \left[\frac{b^{1/3} - (-1)^{1/3} a^{1/3} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}}} \right]}{3 \sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}} (a^2 - b^2)^2 d} - \\
& \frac{2 b^{4/3} (a^2 + 2 b^2) \operatorname{ArcTanh} \left[\frac{b^{1/3} + (-1)^{2/3} a^{1/3} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}}} \right]}{3 \sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}} (a^2 - b^2)^2 d} + \frac{\cos [c+d x]}{12 (a+b) d (1 - \sin [c+d x])^2} + \frac{\cos [c+d x]}{12 (a+b) d (1 - \sin [c+d x])} + \\
& \frac{(a+4b) \cos [c+d x]}{4 (a+b)^2 d (1 - \sin [c+d x])} - \frac{\cos [c+d x]}{12 (a-b) d (1 + \sin [c+d x])^2} - \frac{(a-4b) \cos [c+d x]}{4 (a-b)^2 d (1 + \sin [c+d x])} - \frac{\cos [c+d x]}{12 (a-b) d (1 + \sin [c+d x])}
\end{aligned}$$

Result (type 8, 25 leaves, 0 steps):

$$\text{Unintegrable} \left[\frac{\sec [c+d x]^4}{a+b \sin [c+d x]^3}, x \right]$$

Test results for the 9 problems in "4.1.8 (a+b sin)^m (c+d trig)^n.m"

Test results for the 19 problems in "4.1.9 trig^m (a+b sin^n+c sin^(2 n))^p.m"

Test results for the 294 problems in "4.2.0 (a cos)^m (b trig)^n.m"

Test results for the 189 problems in "4.2.10 (c+d x)^m (a+b cos)^n.m"

Test results for the 62 problems in "4.2.1.1 (a+b cos)^n.m"

Test results for the 99 problems in "4.2.12 (e x)^m (a+b cos(c+d x^n))^p.m"

Test results for the 88 problems in "4.2.1.2 $(g \sin)^p (a+b \cos)^m.m$ "

Test results for the 34 problems in "4.2.13 $(d+e x)^m \cos(a+b x+c x^2)^n.m$ "

Test results for the 22 problems in "4.2.1.3 $(g \tan)^p (a+b \cos)^m.m$ "

Test results for the 932 problems in "4.2.2.1 $(a+b \cos)^m (c+d \cos)^n.m$ "

Test results for the 4 problems in "4.2.2.2 $(g \sin)^p (a+b \cos)^m (c+d \cos)^n.m$ "

Test results for the 1 problems in "4.2.2.3 $(g \cos)^p (a+b \cos)^m (c+d \cos)^n.m$ "

Test results for the 644 problems in "4.2.3.1 $(a+b \cos)^m (c+d \cos)^n (A+B \cos).m$ "

Test results for the 393 problems in "4.2.4.1 $(a+b \cos)^m (A+B \cos+C \cos^2).m$ "

Test results for the 1541 problems in "4.2.4.2 $(a+b \cos)^m (c+d \cos)^n (A+B \cos+C \cos^2).m$ "

Test results for the 98 problems in "4.2.7 $(d \text{ trig})^m (a+b (c \cos)^n)^p.m$ "

Test results for the 21 problems in "4.2.8 $(a+b \cos)^m (c+d \text{ trig})^n.m$ "

Test results for the 20 problems in "4.2.9 $\text{trig}^m (a+b \cos^n+c \cos^{(2 n)})^p.m$ "

Test results for the 387 problems in "4.3.0 $(a \text{ trg})^m (b \tan)^n.m$ "

Test results for the 63 problems in "4.3.10 $(c+d x)^m (a+b \tan)^n.m$ "

Problem 17: Unable to integrate problem.

$$\int \left(\frac{x^2}{\sqrt{\tan[a + b x^2]}} + \frac{\sqrt{\tan[a + b x^2]}}{b} + x^2 \tan[a + b x^2]^{3/2} \right) dx$$

Optimal (type 3, 17 leaves, ? steps):

$$\frac{x \sqrt{\tan[a + b x^2]}}{b}$$

Result (type 8, 55 leaves, 1 step):

$$\text{Unintegrable}\left[\frac{x^2}{\sqrt{\tan[a + b x^2]}}, x\right] + \frac{\text{Unintegrable}\left[\sqrt{\tan[a + b x^2]}, x\right]}{b} + \text{Unintegrable}\left[x^2 \tan[a + b x^2]^{3/2}, x\right]$$

Test results for the 66 problems in "4.3.11 (e x)^m (a+b tan(c+d x^n))^p.m"

Test results for the 700 problems in "4.3.1.2 (d sec)^m (a+b tan)^n.m"

Test results for the 91 problems in "4.3.1.3 (d sin)^m (a+b tan)^n.m"

Test results for the 1328 problems in "4.3.2.1 (a+b tan)^m (c+d tan)^n.m"

Test results for the 855 problems in "4.3.3.1 (a+b tan)^m (c+d tan)^n (A+B tan).m"

Test results for the 171 problems in "4.3.4.2 (a+b tan)^m (c+d tan)^n (A+B tan+C tan^2).m"

Test results for the 499 problems in "4.3.7 (d trig)^m (a+b (c tan)^n)^p.m"

Test results for the 51 problems in "4.3.9 trig^m (a+b tan^n+c tan^(2 n))^p.m"

Test results for the 52 problems in "4.4.0 (a trg)^m (b cot)^n.m"

Test results for the 61 problems in "4.4.10 (c+d x)^m (a+b cot)^n.m"

Test results for the 23 problems in "4.4.1.2 (d csc)^m (a+b cot)^{n.m}"

Test results for the 19 problems in "4.4.1.3 (d cos)^m (a+b cot)^{n.m}"

Test results for the 106 problems in "4.4.2.1 (a+b cot)^m (c+d cot)^{n.m}"

Test results for the 64 problems in "4.4.7 (d trig)^m (a+b (c cot)ⁿ)^{p.m}"

Test results for the 32 problems in "4.4.9 trig^m (a+b cotⁿ+c cot^(2 n))^{p.m}"

Test results for the 299 problems in "4.5.0 (a sec)^m (b trg)^{n.m}"

Test results for the 46 problems in "4.5.10 (c+d x)^m (a+b sec)^{n.m}"

Test results for the 83 problems in "4.5.11 (e x)^m (a+b sec(c+d xⁿ))^{p.m}"

Test results for the 879 problems in "4.5.1.2 (d sec)ⁿ (a+b sec)^{m.m}"

Problem 286: Result unnecessarily involves higher level functions.

$$\int \sec [c + d x]^{5/3} (a + a \sec [c + d x])^{2/3} dx$$

Optimal (type 5, 327 leaves, ? steps):

$$\begin{aligned}
& - \frac{3 a \operatorname{Sec}[c + d x]^{5/3} \operatorname{Sin}[c + d x]}{2 d \left(a \left(1 + \operatorname{Sec}[c + d x]\right)\right)^{1/3}} + \frac{9 \operatorname{Sec}[c + d x]^{2/3} \left(a \left(1 + \operatorname{Sec}[c + d x]\right)\right)^{2/3} \operatorname{Sin}[c + d x]}{4 d} - \frac{9 \left(a \left(1 + \operatorname{Sec}[c + d x]\right)\right)^{2/3} \operatorname{Tan}[c + d x]}{4 d \left(\frac{1}{1 + \operatorname{Cos}[c + d x]}\right)^{1/3} \left(1 + \operatorname{Sec}[c + d x]\right)^{7/3}} + \\
& \left(\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{3}, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^4\right] \left(\operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^4\right)^{1/3} \left(a \left(1 + \operatorname{Sec}[c + d x]\right)\right)^{2/3} \operatorname{Tan}[c + d x] \right) / \\
& \left(8 d \left(\frac{1}{1 + \operatorname{Cos}[c + d x]}\right)^{1/3} \left(1 + \operatorname{Sec}[c + d x]\right)^{4/3} \right) - \\
& \left(5 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{3}{4}, \frac{7}{4}, \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^4\right] \left(\operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^4\right)^{1/3} \left(a \left(1 + \operatorname{Sec}[c + d x]\right)\right)^{2/3} \operatorname{Tan}[c + d x]^3 \right) / \\
& \left(8 d \left(\frac{1}{1 + \operatorname{Cos}[c + d x]}\right)^{1/3} \left(1 + \operatorname{Sec}[c + d x]\right)^{10/3} \right)
\end{aligned}$$

Result (type 6, 79 leaves, 3 steps):

$$\frac{1}{d (1 + \operatorname{Sec}[c + d x])^{7/6}} 2 \times 2^{1/6} \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{2}{3}, -\frac{1}{6}, \frac{3}{2}, 1 - \operatorname{Sec}[c + d x], \frac{1}{2} (1 - \operatorname{Sec}[c + d x])\right] (a + a \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]$$

Test results for the 306 problems in "4.5.1.3 (d sin)^n (a+b sec)^m.m"

Problem 276: Unable to integrate problem.

$$\int \operatorname{Csc}[c + d x]^4 (a + b \operatorname{Sec}[c + d x])^n dx$$

Optimal (type 6, 424 leaves, ? steps):

$$\begin{aligned}
& -\frac{1}{2\sqrt{2}d} \\
& 3 \operatorname{AppellF1}\left[-\frac{1}{2}, \frac{5}{2}, -n, \frac{1}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + dx]), \frac{b (1 - \operatorname{Sec}[c + dx])}{a + b}\right] \operatorname{Cot}[c + dx] \sqrt{1 + \operatorname{Sec}[c + dx]} (a + b \operatorname{Sec}[c + dx])^n \left(\frac{a + b \operatorname{Sec}[c + dx]}{a + b}\right)^{-n} - \\
& \frac{1}{6\sqrt{2}d} \operatorname{AppellF1}\left[-\frac{3}{2}, \frac{5}{2}, -n, -\frac{1}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + dx]), \frac{b (1 - \operatorname{Sec}[c + dx])}{a + b}\right] \operatorname{Cot}[c + dx]^3 \\
& (1 + \operatorname{Sec}[c + dx])^{3/2} (a + b \operatorname{Sec}[c + dx])^n \left(\frac{a + b \operatorname{Sec}[c + dx]}{a + b}\right)^{-n} + \frac{1}{\sqrt{2}d\sqrt{1 + \operatorname{Sec}[c + dx]}} \\
& \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2}, -n, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + dx]), \frac{b (1 - \operatorname{Sec}[c + dx])}{a + b}\right] (a + b \operatorname{Sec}[c + dx])^n \left(\frac{a + b \operatorname{Sec}[c + dx]}{a + b}\right)^{-n} \operatorname{Tan}[c + dx] + \\
& \frac{1}{2\sqrt{2}d\sqrt{1 + \operatorname{Sec}[c + dx]}} \\
& \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -n, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + dx]), \frac{b (1 - \operatorname{Sec}[c + dx])}{a + b}\right] (a + b \operatorname{Sec}[c + dx])^n \left(\frac{a + b \operatorname{Sec}[c + dx]}{a + b}\right)^{-n} \operatorname{Tan}[c + dx]
\end{aligned}$$

Result (type 8, 23 leaves, 0 steps):

$$\text{Unintegrable}[\operatorname{Csc}[c + dx]^4 (a + b \operatorname{Sec}[c + dx])^n, x]$$

Test results for the 365 problems in "4.5.1.4 (d tan)^n (a+b sec)^m.m"

Problem 207: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{Tan}[e + fx]^2}{(a + a \operatorname{Sec}[e + fx])^{9/2}} dx$$

Optimal (type 3, 177 leaves, ? steps):

$$\begin{aligned}
& -\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+a \operatorname{Sec}[e+fx]}}\right]}{a^{9/2} f} + \frac{91 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[e+fx]}}\right]}{32 \sqrt{2} a^{9/2} f} + \\
& \frac{\operatorname{Tan}[e + fx]}{3 a f (a + a \operatorname{Sec}[e + fx])^{7/2}} + \frac{11 \operatorname{Tan}[e + fx]}{24 a^2 f (a + a \operatorname{Sec}[e + fx])^{5/2}} + \frac{27 \operatorname{Tan}[e + fx]}{32 a^3 f (a + a \operatorname{Sec}[e + fx])^{3/2}}
\end{aligned}$$

Result (type 3, 227 leaves, 7 steps):

$$\begin{aligned}
& - \frac{2 \operatorname{ArcTan} \left[\frac{\sqrt{a} \tan[e+f x]}{\sqrt{a+a \sec[e+f x]}} \right]}{a^{9/2} f} + \frac{91 \operatorname{ArcTan} \left[\frac{\sqrt{a} \tan[e+f x]}{\sqrt{2} \sqrt{a+a \sec[e+f x]}} \right]}{32 \sqrt{2} a^{9/2} f} + \frac{27 \sec \left[\frac{1}{2} (e+f x) \right]^2 \sin[e+f x]}{64 a^4 f \sqrt{a+a \sec[e+f x]}} + \\
& \frac{11 \cos[e+f x] \sec \left[\frac{1}{2} (e+f x) \right]^4 \sin[e+f x]}{96 a^4 f \sqrt{a+a \sec[e+f x]}} + \frac{\cos[e+f x]^2 \sec \left[\frac{1}{2} (e+f x) \right]^6 \sin[e+f x]}{24 a^4 f \sqrt{a+a \sec[e+f x]}}
\end{aligned}$$

Problem 347: Unable to integrate problem.

$$\int \frac{(d \tan[e+f x])^n}{a+b \sec[e+f x]} dx$$

Optimal (type 6, 266 leaves, ? steps):

$$\begin{aligned}
& \frac{1}{a f (1-n)} d \operatorname{AppellF1} \left[1-n, \frac{1-n}{2}, \frac{1-n}{2}, 2-n, \frac{a+b}{a+b \sec[e+f x]}, \frac{a-b}{a+b \sec[e+f x]} \right] \left(\frac{b (1-\sec[e+f x])}{a+b \sec[e+f x]} \right)^{\frac{1-n}{2}} \left(\frac{b (1+\sec[e+f x])}{a+b \sec[e+f x]} \right)^{\frac{1-n}{2}} \\
& (d \tan[e+f x])^{-1+n} (-\tan[e+f x]^2)^{\frac{1-n}{2} + \frac{1}{2} (-1+n)} - \frac{d \operatorname{Hypergeometric2F1} \left[1, \frac{1+n}{2}, \frac{3+n}{2}, -\tan[e+f x]^2 \right] (d \tan[e+f x])^{-1+n} (-\tan[e+f x]^2)^{\frac{1-n}{2} + \frac{1+n}{2}}}{a f (1+n)}
\end{aligned}$$

Result (type 8, 25 leaves, 0 steps):

$$\text{Unintegrable} \left[\frac{(d \tan[e+f x])^n}{a+b \sec[e+f x]}, x \right]$$

Test results for the 241 problems in "4.5.2.1 (a+b sec)^m (c+d sec)^n.m"

Problem 217: Unable to integrate problem.

$$\int \frac{(c+d \sec[e+f x])^{3/2}}{\sqrt{a+b \sec[e+f x]}} dx$$

Optimal (type 4, 652 leaves, ? steps):

$$\begin{aligned}
& - \left(\left(2 c (c+d) \operatorname{Cot}[e+f x] \operatorname{EllipticPi} \left[\frac{a (c+d)}{(a+b) c}, \operatorname{ArcSin} \left[\sqrt{\frac{(a+b) (c+d) \operatorname{Sec}[e+f x]}{(c+d) (a+b) \operatorname{Sec}[e+f x]}} \right], \frac{(a-b) (c+d)}{(a+b) (c-d)} \right] \sqrt{\frac{(b c-a d) (1+\operatorname{Sec}[e+f x])}{(c-d) (a+b) \operatorname{Sec}[e+f x]}} \right. \right. \\
& \quad \left. \left. (a+b) \operatorname{Sec}[e+f x] \right)^{3/2} \sqrt{\frac{(a+b) (b c-a d) (-1+\operatorname{Sec}[e+f x]) (c+d) \operatorname{Sec}[e+f x]}{(c+d)^2 (a+b) \operatorname{Sec}[e+f x]^2}} \right) / \left(a (a+b) f \sqrt{c+d} \operatorname{Sec}[e+f x] \right) + \\
& \left(2 d (c+d) \operatorname{Cot}[e+f x] \operatorname{EllipticPi} \left[\frac{b (c+d)}{(a+b) d}, \operatorname{ArcSin} \left[\sqrt{\frac{(a+b) (c+d) \operatorname{Sec}[e+f x]}{(c+d) (a+b) \operatorname{Sec}[e+f x]}} \right], \frac{(a-b) (c+d)}{(a+b) (c-d)} \right] \sqrt{\frac{(b c-a d) (1+\operatorname{Sec}[e+f x])}{(c-d) (a+b) \operatorname{Sec}[e+f x]}} \right. \\
& \quad \left. (a+b) \operatorname{Sec}[e+f x] \right)^{3/2} \sqrt{-\frac{(a+b) (-b c+a d) (-1+\operatorname{Sec}[e+f x]) (c+d) \operatorname{Sec}[e+f x]}{(c+d)^2 (a+b) \operatorname{Sec}[e+f x]^2}} \right) / \left(b (a+b) f \sqrt{c+d} \operatorname{Sec}[e+f x] \right) + \\
& \frac{1}{a b f \sqrt{\frac{(a+b) (c+d) \operatorname{Sec}[e+f x]}{(c+d) (a+b) \operatorname{Sec}[e+f x]}}} 2 (b c-a d) \operatorname{Cot}[e+f x] \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{(a+b) (c+d) \operatorname{Sec}[e+f x]}{(c+d) (a+b) \operatorname{Sec}[e+f x]}} \right], \frac{(a-b) (c+d)}{(a+b) (c-d)} \right] \\
& \quad \sqrt{\frac{(b c-a d) (-1+\operatorname{Sec}[e+f x])}{(c+d) (a+b) \operatorname{Sec}[e+f x]}} \sqrt{\frac{(b c-a d) (1+\operatorname{Sec}[e+f x])}{(c-d) (a+b) \operatorname{Sec}[e+f x]}} \sqrt{a+b} \operatorname{Sec}[e+f x] \sqrt{c+d} \operatorname{Sec}[e+f x]
\end{aligned}$$

Result (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable} \left[\frac{(c+d) \operatorname{Sec}[e+f x]^{3/2}}{\sqrt{a+b} \operatorname{Sec}[e+f x]}, x \right]$$

Test results for the 286 problems in "4.5.2.3 (g sec)^p (a+b sec)^m (c+d sec)ⁿ.m"

Test results for the 634 problems in "4.5.3.1 (a+b sec)^m (d sec)ⁿ (A+B sec).m"

Test results for the 70 problems in "4.5.4.1 (a+b sec)^m (A+B sec+C sec²).m"

Test results for the 1373 problems in "4.5.4.2 (a+b sec)^m (d sec)ⁿ (A+B sec+C sec²).m"

Test results for the 470 problems in "4.5.7 (d trig)^m (a+b (c sec)ⁿ)^p.m"

Problem 132: Unable to integrate problem.

$$\int (a + b \operatorname{Sec}[e + f x]^2)^p (d \operatorname{Sin}[e + f x])^m dx$$

Optimal (type 6, 123 leaves, ? steps):

$$\frac{1}{f(1+m)} \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{1}{2}+p, -p, \frac{3+m}{2}, \operatorname{Sin}[e+f x]^2, \frac{a \operatorname{Sin}[e+f x]^2}{a+b}\right]$$

$$(\operatorname{Cos}[e+f x]^2)^{\frac{1}{2}+p} (a+b \operatorname{Sec}[e+f x]^2)^p (d \operatorname{Sin}[e+f x])^m \left(\frac{a+b-a \operatorname{Sin}[e+f x]^2}{a+b}\right)^{-p} \operatorname{Tan}[e+f x]$$

Result (type 8, 27 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[(a+b \operatorname{Sec}[e+f x]^2)^p (d \operatorname{Sin}[e+f x])^m, x\right]$$

Problem 298: Unable to integrate problem.

$$\int (d \operatorname{Sec}[e+f x])^m (a+b \operatorname{Sec}[e+f x]^2)^p dx$$

Optimal (type 6, 111 leaves, ? steps):

$$\frac{1}{f m} \operatorname{AppellF1}\left[\frac{m}{2}, \frac{1}{2}, -p, \frac{2+m}{2}, \operatorname{Sec}[e+f x]^2, -\frac{b \operatorname{Sec}[e+f x]^2}{a}\right]$$

$$\operatorname{Cot}[e+f x] (d \operatorname{Sec}[e+f x])^m (a+b \operatorname{Sec}[e+f x]^2)^p \left(1+\frac{b \operatorname{Sec}[e+f x]^2}{a}\right)^{-p} \sqrt{-\operatorname{Tan}[e+f x]^2}$$

Result (type 8, 27 leaves, 0 steps):

$$\text{Unintegrable} \left[(\text{d Sec}[e + f x])^m (a + b \text{Sec}[e + f x]^2)^p, x \right]$$

Test results for the 70 problems in "4.6.0 (a csc)^m (b trig)^{n.m}"

Test results for the 84 problems in "4.6.11 (e x)^m (a+b csc(c+d xⁿ))^{p.m}"

Test results for the 59 problems in "4.6.1.2 (d csc)ⁿ (a+b csc)^{m.m}"

Test results for the 16 problems in "4.6.1.3 (d cos)ⁿ (a+b csc)^{m.m}"

Test results for the 23 problems in "4.6.1.4 (d cot)ⁿ (a+b csc)^{m.m}"

Test results for the 24 problems in "4.6.3.1 (a+b csc)^m (d csc)ⁿ (A+B csc).m"

Test results for the 1 problems in "4.6.4.2 (a+b csc)^m (d csc)ⁿ (A+B csc+C csc²).m"

Test results for the 27 problems in "4.6.7 (d trig)^m (a+b (c csc)ⁿ)^{p.m}"

Test results for the 254 problems in "4.7.1 (c trig)^m (d trig)^{n.m}"

Test results for the 294 problems in "4.7.2 trig^m (a trig+b trig)^{n.m}"

Problem 15: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Sin}[x]^3}{(\text{a Cos}[x] + \text{b Sin}[x])^2} dx$$

Optimal (type 3, 107 leaves, ? steps):

$$\frac{6 a^2 b \text{ArcTanh} \left[\frac{-b+a \text{Tan} \left[\frac{x}{2} \right]}{\sqrt{a^2+b^2}} \right]}{\left(a^2+b^2\right)^{5/2}} + \frac{3 a \left(a^2-b^2\right)+a \left(a^2+b^2\right) \text{Cos}[2 x]-b \left(a^2+b^2\right) \text{Sin}[2 x]}{2 \left(a^2+b^2\right)^2 \left(a \text{Cos}[x]+b \text{Sin}[x]\right)}$$

Result (type 3, 283 leaves, 19 steps):

$$\begin{aligned}
& - \frac{3 a^2 \operatorname{ArcTanh} \left[\frac{b \cos[x] - a \sin[x]}{\sqrt{a^2 + b^2}} \right]}{b (a^2 + b^2)^{3/2}} - \frac{2 a^2 b \operatorname{ArcTanh} \left[\frac{b - a \tan \left[\frac{x}{2} \right]}{\sqrt{a^2 + b^2}} \right]}{(a^2 + b^2)^{5/2}} + \frac{2 a^2 (3 a^2 + b^2) \operatorname{ArcTanh} \left[\frac{b - a \tan \left[\frac{x}{2} \right]}{\sqrt{a^2 + b^2}} \right]}{b (a^2 + b^2)^{5/2}} - \frac{\cos[x]}{b^2} + \\
& \frac{3 a^2 \cos[x]}{b^2 (a^2 + b^2)} - \frac{2 a \sin[x]}{b^3} + \frac{3 a^3 \sin[x]}{b^3 (a^2 + b^2)} - \frac{2 a^3 \cos \left[\frac{x}{2} \right]^2 (2 a b + (a^2 - b^2) \tan \left[\frac{x}{2} \right])}{b^3 (a^2 + b^2)^2} + \frac{2 a^2 (a + b \tan \left[\frac{x}{2} \right])}{(a^2 + b^2)^2 (a + 2 b \tan \left[\frac{x}{2} \right] - a \tan \left[\frac{x}{2} \right]^2)}
\end{aligned}$$

Problem 23: Result valid but suboptimal antiderivative.

$$\int \frac{\sin[x]^2}{(a \cos[x] + b \sin[x])^3} dx$$

Optimal (type 3, 92 leaves, ? steps):

$$\begin{aligned}
& - \frac{(a^2 - 2 b^2) \operatorname{ArcTanh} \left[\frac{-b + a \tan \left[\frac{x}{2} \right]}{\sqrt{a^2 + b^2}} \right]}{(a^2 + b^2)^{5/2}} + \frac{a (3 a b \cos[x] + (a^2 + 4 b^2) \sin[x])}{2 (a^2 + b^2)^2 (a \cos[x] + b \sin[x])^2}
\end{aligned}$$

Result (type 3, 300 leaves, 13 steps):

$$\begin{aligned}
& \frac{2 a^2 \operatorname{ArcTanh} \left[\frac{b \cos[x] - a \sin[x]}{\sqrt{a^2 + b^2}} \right]}{b^2 (a^2 + b^2)^{3/2}} - \frac{\operatorname{ArcTanh} \left[\frac{b \cos[x] - a \sin[x]}{\sqrt{a^2 + b^2}} \right]}{b^2 \sqrt{a^2 + b^2}} - \frac{a^2 (2 a^2 - b^2) \operatorname{ArcTanh} \left[\frac{b - a \tan \left[\frac{x}{2} \right]}{\sqrt{a^2 + b^2}} \right]}{b^2 (a^2 + b^2)^{5/2}} + \\
& \frac{2 a}{b (a^2 + b^2) (a \cos[x] + b \sin[x])} + \frac{2 (a b + (a^2 + 2 b^2) \tan \left[\frac{x}{2} \right])}{a (a^2 + b^2) \left(a + 2 b \tan \left[\frac{x}{2} \right] - a \tan \left[\frac{x}{2} \right]^2 \right)^2} - \frac{4 a^4 + 3 a^2 b^2 + 2 b^4 + a b (5 a^2 + 2 b^2) \tan \left[\frac{x}{2} \right]}{a b (a^2 + b^2)^2 \left(a + 2 b \tan \left[\frac{x}{2} \right] - a \tan \left[\frac{x}{2} \right]^2 \right)}
\end{aligned}$$

Problem 123: Result valid but suboptimal antiderivative.

$$\int \frac{\cos[c + d x]^3}{(a \cos[c + d x] + b \sin[c + d x])^2} dx$$

Optimal (type 3, 138 leaves, ? steps):

$$\begin{aligned}
& - \frac{3 a b^2 \operatorname{ArcTanh} \left[\frac{b \cos[c + d x] - a \sin[c + d x]}{\sqrt{a^2 + b^2}} \right]}{(a^2 + b^2)^{5/2} d} + \frac{2 a b \cos[c + d x]}{(a^2 + b^2)^2 d} + \frac{(a^2 - b^2) \sin[c + d x]}{(a^2 + b^2)^2 d} - \frac{b^3}{(a^2 + b^2)^2 d (a \cos[c + d x] + b \sin[c + d x])}
\end{aligned}$$

Result (type 3, 231 leaves, 11 steps):

$$\begin{aligned} & \frac{2 b^4 \operatorname{Arctanh}\left[\frac{b-a \tan\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a(a^2+b^2)^{5/2} d}-\frac{2 b^2(3 a^2+b^2) \operatorname{Arctanh}\left[\frac{b-a \tan\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a(a^2+b^2)^{5/2} d}+ \\ & \frac{2(2 a b+(a^2-b^2) \tan\left[\frac{1}{2}(c+d x)\right])}{(a^2+b^2)^2 d\left(1+\tan\left[\frac{1}{2}(c+d x)\right]^2\right)}-\frac{2 b^3\left(a+b \tan\left[\frac{1}{2}(c+d x)\right]\right)}{a(a^2+b^2)^2 d\left(a+2 b \tan\left[\frac{1}{2}(c+d x)\right]-a \tan\left[\frac{1}{2}(c+d x)\right]^2\right)} \end{aligned}$$

Problem 131: Result valid but suboptimal antiderivative.

$$\int \frac{\cos [c+d x]^4}{(a \cos [c+d x]+b \sin [c+d x])^3} d x$$

Optimal (type 3, 216 leaves, ? steps):

$$\begin{aligned} & -\frac{3 b^2(4 a^2-b^2) \operatorname{Arctanh}\left[\frac{b-a \tan\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{(a^2+b^2)^{7/2} d}+\frac{b(3 a^2-b^2) \cos [c+d x]}{(a^2+b^2)^3 d}+\frac{a(a^2-3 b^2) \sin [c+d x]}{(a^2+b^2)^3 d}+ \\ & \frac{b^4 \sin [c+d x]}{2 a(a^2+b^2)^2 d\left(a \cos [c+d x]+b \sin [c+d x]\right)^2}-\frac{b^3(8 a^2+b^2)}{2 a(a^2+b^2)^3 d\left(a \cos [c+d x]+b \sin [c+d x]\right)} \end{aligned}$$

Result (type 3, 492 leaves, 15 steps):

$$\begin{aligned} & -\frac{3 b^4(a^2+2 b^2) \operatorname{Arctanh}\left[\frac{b-a \tan\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a^2(a^2+b^2)^{7/2} d}+\frac{4 b^4(3 a^2+2 b^2) \operatorname{Arctanh}\left[\frac{b-a \tan\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a^2(a^2+b^2)^{7/2} d}-\frac{2 b^2(6 a^4+3 a^2 b^2+b^4) \operatorname{Arctanh}\left[\frac{b-a \tan\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a^2(a^2+b^2)^{7/2} d}+ \\ & \frac{2(b(3 a^2-b^2)+a(a^2-3 b^2) \tan\left[\frac{1}{2}(c+d x)\right])}{(a^2+b^2)^3 d\left(1+\tan\left[\frac{1}{2}(c+d x)\right]^2\right)}+\frac{2 b^4\left(a b+(a^2+2 b^2) \tan\left[\frac{1}{2}(c+d x)\right]\right)}{a^3(a^2+b^2)^2 d\left(a+2 b \tan\left[\frac{1}{2}(c+d x)\right]-a \tan\left[\frac{1}{2}(c+d x)\right]^2\right)}- \\ & \frac{3 b^4(a^2+2 b^2)\left(b-a \tan\left[\frac{1}{2}(c+d x)\right]\right)}{a^3(a^2+b^2)^3 d\left(a+2 b \tan\left[\frac{1}{2}(c+d x)\right]-a \tan\left[\frac{1}{2}(c+d x)\right]^2\right)}-\frac{4 b^3\left(2 a^4-b^4+a b(3 a^2+2 b^2) \tan\left[\frac{1}{2}(c+d x)\right]\right)}{a^3(a^2+b^2)^3 d\left(a+2 b \tan\left[\frac{1}{2}(c+d x)\right]-a \tan\left[\frac{1}{2}(c+d x)\right]^2\right)} \end{aligned}$$

Problem 133: Result valid but suboptimal antiderivative.

$$\int \frac{\cos [c+d x]^2}{(a \cos [c+d x]+b \sin [c+d x])^3} d x$$

Optimal (type 3, 119 leaves, ? steps):

$$\frac{(2 a^2 - b^2) \operatorname{ArcTanh} \left[\frac{-b+a \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^2+b^2}} \right]}{(a^2+b^2)^{5/2} d} - \frac{b \left((4 a^2 + b^2) \cos [c+d x] + 3 a b \sin [c+d x] \right)}{2 (a^2+b^2)^2 d (a \cos [c+d x] + b \sin [c+d x])^2}$$

Result (type 3, 225 leaves, 6 steps):

$$-\frac{(2 a^2 - b^2) \operatorname{ArcTanh} \left[\frac{b-a \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^2+b^2}} \right]}{(a^2+b^2)^{5/2} d} + \frac{2 b^2 \left(a b + (a^2 + 2 b^2) \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right] \right)}{a^3 (a^2+b^2) d \left(a + 2 b \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right] - a \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]^2 \right)^2} -$$

$$\frac{b \left(4 a^4 + 3 a^2 b^2 + 2 b^4 + a b (5 a^2 + 2 b^2) \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right] \right)}{a^3 (a^2+b^2)^2 d \left(a + 2 b \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right] - a \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]^2 \right)}$$

Problem 142: Result valid but suboptimal antiderivative.

$$\int \frac{\cos [c+d x]^3}{(a \cos [c+d x] + b \sin [c+d x])^4} dx$$

Optimal (type 3, 157 leaves, ? steps):

$$\frac{a (2 a^2 - 3 b^2) \operatorname{ArcTanh} \left[\frac{-b+a \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^2+b^2}} \right]}{(a^2+b^2)^{7/2} d} + \frac{-3 (3 a^4 b - a^2 b^3 + b^5) \cos [2 (c+d x)] + \frac{1}{2} b (-9 a^2 + b^2) (2 (a^2 + b^2) + 3 a b \sin [2 (c+d x)])}{6 (a^2+b^2)^3 d (a \cos [c+d x] + b \sin [c+d x])^3}$$

Result (type 3, 362 leaves, 7 steps):

$$-\frac{a (2 a^2 - 3 b^2) \operatorname{ArcTanh} \left[\frac{b-a \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^2+b^2}} \right]}{(a^2+b^2)^{7/2} d} - \frac{8 b^3 \left(a (a^2 + 2 b^2) + b (3 a^2 + 4 b^2) \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right] \right)}{3 a^5 (a^2+b^2) d \left(a + 2 b \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right] - a \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]^2 \right)^3} +$$

$$\frac{2 b^2 \left(b (15 a^4 + 18 a^2 b^2 + 8 b^4) + a (9 a^4 + 30 a^2 b^2 + 16 b^4) \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right] \right)}{3 a^5 (a^2+b^2)^2 d \left(a + 2 b \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right] - a \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]^2 \right)^2} -$$

$$\frac{b \left(6 a^6 + 9 a^4 b^2 + 12 a^2 b^4 + 4 b^6 + a b (9 a^4 + 6 a^2 b^2 + 2 b^4) \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right] \right)}{a^4 (a^2+b^2)^3 d \left(a + 2 b \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right] - a \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]^2 \right)}$$

Test results for the 397 problems in "4.7.3 (c+d x)^m trig^n trig^p.m"

Test results for the 9 problems in "4.7.4 $x^m (a+b \operatorname{trig}^n)^p.m"$

Test results for the 330 problems in "4.7.5 $x^m \operatorname{trig}(a+b \operatorname{log}(c x^n))^p.m"$

Problem 259: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(- (1 + b^2 n^2) \operatorname{Sec} [a + b \operatorname{Log} [c x^n]] + 2 b^2 n^2 \operatorname{Sec} [a + b \operatorname{Log} [c x^n]]^3 \right) dx$$

Optimal (type 3, 41 leaves, ? steps):

$$-x \operatorname{Sec} [a + b \operatorname{Log} [c x^n]] + b n x \operatorname{Sec} [a + b \operatorname{Log} [c x^n]] \operatorname{Tan} [a + b \operatorname{Log} [c x^n]]$$

Result (type 5, 175 leaves, 7 steps):

$$\frac{-2 e^{i a} (1 - \frac{i}{b n}) x (c x^n)^{\frac{i}{b}} \operatorname{Hypergeometric2F1} [1, \frac{1}{2} \left(1 - \frac{i}{b n}\right), \frac{1}{2} \left(3 - \frac{i}{b n}\right), -e^{2 i a} (c x^n)^{2 \frac{i}{b}}] + 16 b^2 e^{3 i a} n^2 x (c x^n)^{3 \frac{i}{b}} \operatorname{Hypergeometric2F1} [3, \frac{1}{2} \left(3 - \frac{i}{b n}\right), \frac{1}{2} \left(5 - \frac{i}{b n}\right), -e^{2 i a} (c x^n)^{2 \frac{i}{b}}]}{1 + 3 \frac{i}{b n}}$$

Problem 260: Result unnecessarily involves higher level functions.

$$\int x^m \operatorname{Sec} [a + 2 \operatorname{Log} [c x^{\frac{1}{2} \sqrt{-(1+m)^2}}]]^3 dx$$

Optimal (type 3, 110 leaves, ? steps):

$$\frac{x^{1+m} \operatorname{Sec} [a + 2 \operatorname{Log} [c x^{\frac{1}{2} \sqrt{-(1+m)^2}}]]}{2 (1+m)} + \frac{x^{1+m} \operatorname{Sec} [a + 2 \operatorname{Log} [c x^{\frac{1}{2} \sqrt{-(1+m)^2}}]] \operatorname{Tan} [a + 2 \operatorname{Log} [c x^{\frac{1}{2} \sqrt{-(1+m)^2}}]]}{2 \sqrt{-(1+m)^2}}$$

Result (type 5, 146 leaves, 3 steps):

$$\left(\frac{8 e^{3 i a} x^{1+m} \left(c x^{\frac{1}{2} \sqrt{-(1+m)^2}}\right)^6 \operatorname{Hypergeometric2F1} [3, \frac{1}{2} \left(3 - \frac{i (1+m)}{\sqrt{-(1+m)^2}}\right), \frac{1}{2} \left(5 - \frac{i (1+m)}{\sqrt{-(1+m)^2}}\right), -e^{2 i a} \left(c x^{\frac{1}{2} \sqrt{-(1+m)^2}}\right)^4]}{\left(1 - \frac{i}{m} (m - 3 \sqrt{-(1+m)^2}\right)} \right)$$

Problem 301: Result unnecessarily involves higher level functions and more than twice size of optimal

antiderivative.

$$\int \left(- (1 + b^2 n^2) \csc[a + b \log[c x^n]] + 2 b^2 n^2 \csc[a + b \log[c x^n]]^3 \right) dx$$

Optimal (type 3, 42 leaves, ? steps):

$$-x \csc[a + b \log[c x^n]] - b n x \cot[a + b \log[c x^n]] \csc[a + b \log[c x^n]]$$

Result (type 5, 172 leaves, 7 steps):

$$\begin{aligned} & 2 e^{i a} (i + b n) x (c x^n)^{i b} \text{Hypergeometric2F1}\left[1, \frac{1}{2} \left(1 - \frac{i}{b n}\right), \frac{1}{2} \left(3 - \frac{i}{b n}\right), e^{2 i a} (c x^n)^{2 i b}\right] - \\ & \frac{16 b^2 e^{3 i a} n^2 x (c x^n)^{3 i b} \text{Hypergeometric2F1}\left[3, \frac{1}{2} \left(3 - \frac{i}{b n}\right), \frac{1}{2} \left(5 - \frac{i}{b n}\right), e^{2 i a} (c x^n)^{2 i b}\right]}{i - 3 b n} \end{aligned}$$

Problem 302: Result unnecessarily involves higher level functions.

$$\int x^m \csc[a + 2 \log[c x^{\frac{1}{2} \sqrt{-(1+m)^2} }]^3] dx$$

Optimal (type 3, 110 leaves, ? steps):

$$\frac{x^{1+m} \csc[a + 2 \log[c x^{\frac{1}{2} \sqrt{-(1+m)^2} }]]}{2 (1+m)} - \frac{x^{1+m} \cot[a + 2 \log[c x^{\frac{1}{2} \sqrt{-(1+m)^2} }]] \csc[a + 2 \log[c x^{\frac{1}{2} \sqrt{-(1+m)^2} }]]}{2 \sqrt{-(1+m)^2}}$$

Result (type 5, 142 leaves, 3 steps):

$$-\frac{1}{i + i m - 3 \sqrt{-(1+m)^2}} 8 e^{3 i a} x^{1+m} \left(c x^{\frac{1}{2} \sqrt{-(1+m)^2}}\right)^{6 i} \text{Hypergeometric2F1}\left[3, \frac{1}{2} \left(3 - \frac{i (1+m)}{\sqrt{-(1+m)^2}}\right), \frac{1}{2} \left(5 - \frac{i (1+m)}{\sqrt{-(1+m)^2}}\right), e^{2 i a} \left(c x^{\frac{1}{2} \sqrt{-(1+m)^2}}\right)^{4 i}\right]$$

Test results for the 142 problems in "4.7.6 f^(a+b x+c x^2) trig(d+e x+f x^2)^n.m"

Problem 28: Unable to integrate problem.

$$\int F^c (a+b x) (f x)^m \sin[d + e x] dx$$

Optimal (type 4, 139 leaves, ? steps):

$$\frac{e^{-i d} F^a c \ (f x)^m \text{Gamma}[1+m, x (\frac{i}{\pi} e - b c \text{Log}[F])] \ (x (\frac{i}{\pi} e - b c \text{Log}[F]))^{-m}}{2 (e + \frac{i}{\pi} b c \text{Log}[F])} -$$

$$\frac{e^{i d} F^a c \ (f x)^m \text{Gamma}[1+m, -x (\frac{i}{\pi} e + b c \text{Log}[F])] \ (-x (\frac{i}{\pi} e + b c \text{Log}[F]))^{-m}}{2 (e - \frac{i}{\pi} b c \text{Log}[F])}$$

Result (type 8, 24 leaves, 1 step):

$$\text{CannotIntegrate}[F^{a+c+b c x} (f x)^m \sin[d+e x], x]$$

Problem 32: Unable to integrate problem.

$$\int f F^{c(a+b x)} (f x)^m (e x \cos[d+e x] + (1+m+b c x \text{Log}[F]) \sin[d+e x]) dx$$

Optimal (type 3, 23 leaves, ? steps):

$$f F^{c(a+b x)} x (f x)^m \sin[d+e x]$$

Result (type 8, 89 leaves, 6 steps):

$$e \text{CannotIntegrate}[F^{a+c+b c x} (f x)^{1+m} \cos[d+e x], x] +$$

$$f (1+m) \text{CannotIntegrate}[F^{a+c+b c x} (f x)^m \sin[d+e x], x] + b c \text{CannotIntegrate}[F^{a+c+b c x} (f x)^{1+m} \sin[d+e x], x] \text{Log}[F]$$

Test results for the 950 problems in "4.7.7 Trig functions.m"

Problem 759: Result valid but suboptimal antiderivative.

$$\int (\cos[x]^{12} \sin[x]^{10} - \cos[x]^{10} \sin[x]^{12}) dx$$

Optimal (type 3, 12 leaves, ? steps):

$$\frac{1}{11} \cos[x]^{11} \sin[x]^{11}$$

Result (type 3, 129 leaves, 25 steps):

$$\frac{3 \cos[x]^{11} \sin[x]}{5632} - \frac{3 \cos[x]^{13} \sin[x]}{5632} + \frac{1}{512} \cos[x]^{11} \sin[x]^3 - \frac{7 \cos[x]^{13} \sin[x]^3}{2816} + \frac{7 \cos[x]^{11} \sin[x]^5}{1280} - \frac{7}{880} \cos[x]^{13} \sin[x]^5 +$$

$$\frac{1}{80} \cos[x]^{11} \sin[x]^7 - \frac{9}{440} \cos[x]^{13} \sin[x]^7 + \frac{1}{40} \cos[x]^{11} \sin[x]^9 - \frac{1}{22} \cos[x]^{13} \sin[x]^9 + \frac{1}{22} \cos[x]^{11} \sin[x]^{11}$$

Problem 796: Unable to integrate problem.

$$\int e^{\sin[x]} \sec[x]^2 (x \cos[x]^3 - \sin[x]) dx$$

Optimal (type 3, 13 leaves, ? steps):

$$e^{\sin[x]} (-1 + x \cos[x]) \sec[x]$$

Result (type 8, 24 leaves, 2 steps):

$$\text{CannotIntegrate}[e^{\sin[x]} x \cos[x], x] - \text{CannotIntegrate}[e^{\sin[x]} \sec[x] \tan[x], x]$$

Problem 858: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\cos[x]^{3/2} \sqrt{3 \cos[x] + \sin[x]}} dx$$

Optimal (type 3, 19 leaves, ? steps):

$$\frac{2 \sqrt{3 \cos[x] + \sin[x]}}{\sqrt{\cos[x]}}$$

Result (type 3, 88 leaves, 5 steps):

$$\frac{2 \cos[\frac{x}{2}]^2 (3 + 2 \tan[\frac{x}{2}] - 3 \tan[\frac{x}{2}]^2)}{\sqrt{\cos[\frac{x}{2}]^2 (3 + 2 \tan[\frac{x}{2}] - 3 \tan[\frac{x}{2}]^2)} \sqrt{\cos[\frac{x}{2}]^2 (1 - \tan[\frac{x}{2}]^2)}}$$

Problem 859: Unable to integrate problem.

$$\int \frac{\csc[x] \sqrt{\cos[x] + \sin[x]}}{\cos[x]^{3/2}} dx$$

Optimal (type 3, 44 leaves, ? steps):

$$-\text{Log}[\sin[x]] + 2 \text{Log}[-\sqrt{\cos[x]} + \sqrt{\cos[x] + \sin[x]}] + \frac{2 \sqrt{\cos[x] + \sin[x]}}{\sqrt{\cos[x]}}$$

Result (type 8, 20 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\csc[x] \sqrt{\cos[x] + \sin[x]}}{\cos[x]^{3/2}}, x\right]$$

Problem 860: Result valid but suboptimal antiderivative.

$$\int \frac{\cos[x] + \sin[x]}{\sqrt{1 + \sin[2x]}} dx$$

Optimal (type 3, 19 leaves, ? steps):

$$\underline{x\sqrt{1+\sin[2x]}}$$

$$\cos[x] + \sin[x]$$

Result (type 3, 72 leaves, 17 steps):

$$\frac{2 \operatorname{ArcTan}[\tan[\frac{x}{2}]] \cos[\frac{x}{2}]^2 (1 + 2 \tan[\frac{x}{2}] - \tan[\frac{x}{2}]^2)}{\sqrt{\cos[\frac{x}{2}]^4 (1 + 2 \tan[\frac{x}{2}] - \tan[\frac{x}{2}]^2)^2}}$$

Problem 912: Result valid but suboptimal antiderivative.

$$\int \frac{\cos[x] + \sin[x]}{\sqrt{\cos[x]}\sqrt{\sin[x]}} dx$$

Optimal (type 3, 57 leaves, ? steps):

$$-\sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right] + \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]$$

Result (type 3, 243 leaves, 22 steps):

$$\begin{aligned} & \frac{\operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\cos[x]}}{\sqrt{\sin[x]}}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\cos[x]}}{\sqrt{\sin[x]}}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]}{\sqrt{2}} + \frac{\operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]}{\sqrt{2}} - \\ & \frac{\log\left[1 + \cot[x] - \frac{\sqrt{2} \sqrt{\cos[x]}}{\sqrt{\sin[x]}}\right]}{2\sqrt{2}} + \frac{\log\left[1 + \cot[x] + \frac{\sqrt{2} \sqrt{\cos[x]}}{\sqrt{\sin[x]}}\right]}{2\sqrt{2}} + \frac{\log\left[1 - \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}} + \tan[x]\right]}{2\sqrt{2}} - \frac{\log\left[1 + \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}} + \tan[x]\right]}{2\sqrt{2}} \end{aligned}$$

Problem 914: Unable to integrate problem.

$$\int (10x^9 \cos[x^5 \log[x]] - x^{10} (x^4 + 5x^4 \log[x]) \sin[x^5 \log[x]]) dx$$

Optimal (type 3, 11 leaves, ? steps):

$$x^{10} \cos[x^5 \log[x]]$$

Result (type 8, 48 leaves, 4 steps):

$$10 \text{CannotIntegrate}[x^9 \cos[x^5 \log[x]], x] - \text{CannotIntegrate}[x^{14} \sin[x^5 \log[x]], x] - 5 \text{CannotIntegrate}[x^{14} \log[x] \sin[x^5 \log[x]], x]$$

Problem 915: Unable to integrate problem.

$$\int \cos\left[\frac{x}{2}\right]^2 \tan\left[\frac{\pi}{4} + \frac{x}{2}\right] dx$$

Optimal (type 3, 27 leaves, ? steps):

$$\frac{x}{2} - \frac{\cos[x]}{2} - \log[\cos\left[\frac{\pi}{4} + \frac{x}{2}\right]]$$

Result (type 8, 23 leaves, 0 steps):

$$\text{CannotIntegrate}[\cos\left[\frac{x}{2}\right]^2 \tan\left[\frac{\pi}{4} + \frac{x}{2}\right], x]$$

Problem 931: Unable to integrate problem.

$$\int \left(\frac{x^4}{b \sqrt{x^3 + 3 \sin[a + b x]}} + \frac{x^2 \cos[a + b x]}{\sqrt{x^3 + 3 \sin[a + b x]}} + \frac{4 x \sqrt{x^3 + 3 \sin[a + b x]}}{3 b} \right) dx$$

Optimal (type 3, 26 leaves, ? steps):

$$\frac{2 x^2 \sqrt{x^3 + 3 \sin[a + b x]}}{3 b}$$

Result (type 8, 82 leaves, 1 step):

$$\frac{\text{CannotIntegrate}\left[\frac{x^4}{\sqrt{x^3 + 3 \sin[a + b x]}}, x\right]}{b} + \text{CannotIntegrate}\left[\frac{x^2 \cos[a + b x]}{\sqrt{x^3 + 3 \sin[a + b x]}}, x\right] + \frac{4 \text{CannotIntegrate}\left[x \sqrt{x^3 + 3 \sin[a + b x]}, x\right]}{3 b}$$

Problem 933: Unable to integrate problem.

$$\int \frac{\cos[x] + \sin[x]}{e^{-x} + \sin[x]} dx$$

Optimal (type 3, 9 leaves, ? steps):

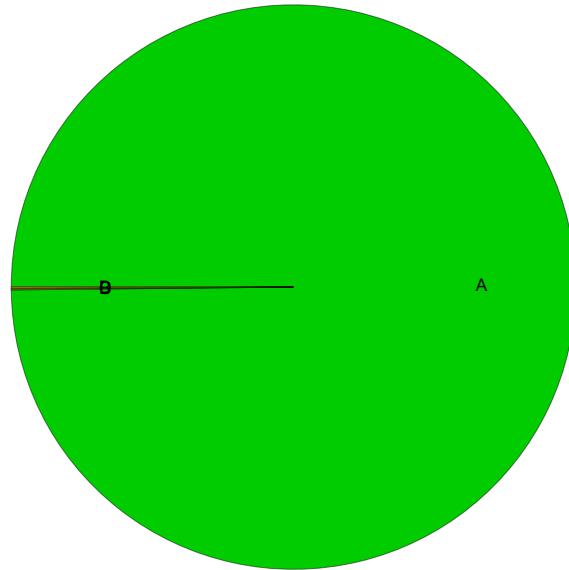
$$\log[1 + e^x \sin[x]]$$

Result (type 8, 36 leaves, 5 steps):

$$x - \text{CannotIntegrate}\left[\frac{1}{1 + e^x \sin[x]}, x\right] - \text{CannotIntegrate}\left[\frac{\cot[x]}{1 + e^x \sin[x]}, x\right] + \log[\sin[x]]$$

Summary of Integration Test Results

22551 integration problems



- A - 22515 optimal antiderivatives
- B - 12 valid but suboptimal antiderivatives
- C - 5 unnecessarily complex antiderivatives
- D - 19 unable to integrate problems
- E - 0 integration timeouts
- F - 0 invalid antiderivatives